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Soliton Solutions for some Higher-Order Modified KDV Equations

Azza AM Abd Elatif[1*]
Ahmed S Abd Elradi[1]
Mohammed K Elboree[1]
Fawzi Abdelati[1]

[1]Department of Mathematics, Faculty of Science, South Valley University, Qena, Egypt

Abstract

In this article, we apply a traditional method to find the soliton solutions for some nonlinear evolution equations in mathematical physics, the tanh-method and the exp-function method with the aid of computer software Mathematica. We apply the tanh-method and exp-function to construct the exact traveling wave solutions of the nonlinear modified kdv fifth-order equation and the kudryashov method with the nonlinear modified seventh-order equation. These equations have wide applications electro-magnetic waves in size quantized films, traffic flow and elastic media.

Keywords: The tanh-method, The exp-function method, Nonlinear evolution equations, Soliton solutions, The kudryashov method, Homogenous balance method.

Introduction

Nonlinear phenomena plays a fundamental role in applied mathematics and physics. It is known that the best models for representation of nonlinear phenomena are the kdv equations.

It is known that the third order kdv equation is the common model for studying weakly nonlinear waves. However several other extension of the of standard kdv equations like modified kdv appears in scientific applications. The modified kdv equation differs from the original kdv equation in the nonlinear term only, where it includes u^2u_x instead of uu_x but both include the dispersion term u_{xxx}. This change in the nonlinear terms causes several substantial differences in the structures of the solutions. However, the kdv and mkdv equations are linked at a deeper level by the so called Miura transformation [3].

$$u = v^2 + v_x \tag{1}$$

Is like the kdv equation that the modified equation appears in higher order versions as well.

Modified kdv equations of fifth-order and seventh-order were formally derived [3].

$$u_t + \{6u^5 + \sigma\left(u u_x^{\ 2} + u^2 u_{2x}\right) + u_{4x}\}_x = 0 \tag{2}$$

$$u_t + \{20u^7 + 70\left(u^2 u_{2x} + 2u^3 u_x^{\ 2} + 14\left(u^2 u_{4x} + 3uu_{2x}^{\ 2} + 4uu_x u_{3x} + 5u_x^{\ 2} u_{2x}\right) + u_{6x}\right)\}_x = 0 \tag{3}$$

The goal of this paper finding new soliton solutions and periodic solutions for nonlinear modified higher order kdv equations by using powerful methods in solving nonlinear evolution equations as tanh-method and exp-function method. The remainder of the paper is organized as follows:

In section 2, a brief discussion for the tanh-method and exp-function method are presented and soliton solutions of fifth-order mkdv

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***Corresponding author:** Dr. Azza AM Abd Elatif, Department of Mathematics, Faculty of Science, South Valley University, Qena, Egypt. E-mail: azaali27@yahoo.com

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equations. In section 3 we describe the kudryshov method and apply to seventh-order mkdv equations. In section 4 Physical explanations of our obtained solutions. In section 5 conclusion and References.

The Tanh-Method

The tanh-method [2] is a powerful solution method for computation of exact traveling wave solutions. Various extension forms of the tanh-method have been developed [2]. First a power series in tanh was used as ansatz to obtain analytical solutions of traveling wave type of certain nonlinear evolution equations. To avoid complexity Malfiet had customized the tanh Technique by introducing tanh as a new variable, since all derivatives of tanh are represented by a tanh itself. A Mathmatica package deals with the tedious algebra that arises from using the tanh- method and output directly the required solutions. We will use the tanh-method which is standard form for solving the fifth-order mkdv and seventh-order mkdv. The main steps of the standard tanh-method are as follows: -

a) We first consider a general form of nonlinear equation

$$p\left(u, u_t, u_x, u_{xx}, u_{tt}, \ldots\ldots\ldots\right)=0 \tag{4}$$

To find the traveling wave solution of equations (2), (3) we introduce the wave variable = x-ct so that

$$u\left(x, t\right)=u(\xi) \tag{5}$$

Where the localized wave solution $u(\xi)$ travels with speed c. Based on this we use the following changes

$$\frac{\partial}{\partial t}=-c\frac{d}{d\xi}$$

$$\frac{\partial}{\partial x}=\frac{d}{d\xi}$$

$$\frac{\partial 2}{\partial x^2}=\frac{d2}{d\xi^2} \tag{6}$$

$$\frac{\partial 3}{\partial x^3}=\frac{d3}{d\xi^3}$$

And so on for other derivatives using (5) changes the PDE (4) to an ODE

$$p(u, u', u'', \ldots\ldots\ldots\ldots)=0 \tag{7}$$

a) If all terms of the resulting ODE contain derivatives in x, then introduce a new independent variable

$$\sqrt{b}\operatorname{Tanh}\sqrt{b}\xi=y \tag{8}$$

That leads to the change of derivatives :

$$\frac{d}{d\xi}=\sqrt{b}\left(1-y^2\right)\frac{d}{dy}$$

$$\frac{d2}{d\xi^2}=\sqrt{b}^2\left(1-y^2\right)(-2y\frac{d}{dy}+\left(1-y^2\right)\frac{d2}{dy^2}) \tag{9}$$

$$\frac{d3}{d\xi^3}=2\sqrt{b}^3\left(1-y^2\right)\left(3y^2-1\right)\frac{d}{dy}-6\sqrt{b}^3y(1-y^2)^2\frac{d2}{dy^2}+\sqrt{b}^3(1-y^2)^3\frac{d3}{dy^3}$$

Where other derivatives can be derived in a similar manner

Introduce the ansatz

$$u\left(\xi\right)=s\left(y\right)=\sum_{k=0}^{M}a_k y^k \tag{10}$$

Where M is a positive integer, in most cases, that will be determined. Substituting Eq (9) and Eq (10) in to the ODE Eq (7) yields an equation in powers of y.

To determine the parameter M

We usually balance the linear of highest order in the resulting equation. This will give a system of algebraic equations involving the a_k, (k=0,1,………,M), M and c. Having determined these parameters, knowing that M is appositive integer in most cases, and using Eq (10) we obtain an analytic solution in a closed form.

Using tanh-method for the nonlinear fifth-modified KDV

We know that the nonlinear fifth-modified kdv equation has formula: -

$$u_t+\{6u^5+\sigma\left(u u_x{}^2+u^2u_{2x}\right)+u_{4x}\}_x=0 \tag{11}$$

Let σ =1 and using wave variable ξ =x-ct Eq (11) becomes ODE

$$-cu'+\{6u^5+\left(uu'+u^2u''\right)+u^{(4)}\}'=0 \tag{12}$$

Let $u\left(\xi\right)=\sum_{k=0}^{M}a_k y^k$ (13)

By balancing the linear terms of highest order with highest order nonlinear terms in Eq (13) we get M=1 this leads to:

$$u\left(\xi\right)=a_0+a_1y \tag{14}$$

$$y'=b+y^2 \tag{15}$$

Where b is parameter substituting Eq (12) into Eq (11), collecting the coefficients of each power of y, setting each coefficient to zero and solving resulting system of algebraic equations we find the following sets of solutions :-

$$a_0=0, a_1=-\sqrt{-\frac{5}{3}-\frac{i\sqrt{11}}{3}}, b=\frac{1}{24}(-\sqrt{3}\left(-5-i\sqrt{11}\right)^{\frac{3}{2}}-4\sqrt{3\left(-5-i\sqrt{11}\right)}, c=\frac{1}{2}(-7-5i\sqrt{11}) \tag{16}$$

$$a_0=0, a_1=\sqrt{-\frac{5}{3}-\frac{i\sqrt{11}}{3}}, b=\frac{1}{24}\left(\sqrt{3}\left(-5-i\sqrt{11}\right)^{\frac{3}{2}}+4\sqrt{3\left(-5-i\sqrt{11}\right)}\right), c=\frac{1}{2}(-7-5i\sqrt{11}) \tag{17}$$

$$a_0=0, a_1=-\sqrt{-\frac{5}{3}+\frac{i\sqrt{11}}{3}}, b=\frac{1}{24}\left(-\sqrt{3}\left(-5+i\sqrt{11}\right)^{\frac{3}{2}}-4\sqrt{3\left(-5+i\sqrt{11}\right)}\right), c=\frac{1}{2}(-7+5i\sqrt{11}) \tag{18}$$

$$a_0=0, a_1=\sqrt{-\frac{5}{3}+\frac{i\sqrt{11}}{3}}, b=\frac{1}{24}\left(\sqrt{3}\left(-5+i\sqrt{11}\right)^{\frac{3}{2}}+4\sqrt{3\left(-5+i\sqrt{11}\right)}\right), c=\frac{1}{2}\left(-7+5i\sqrt{11}\right) \tag{19}$$

According to equation Eq (16) where b<0 [4],

$$u\left(x, t\right)=\sqrt{-\frac{5}{3}-\frac{i\sqrt{11}}{3}}\operatorname{Tanh}\left[\frac{1}{2}(-7-5i\sqrt{11})t-x\right] \tag{20}$$

Where b>0

$$u\left(x, t\right)=\sqrt{-\frac{5}{3}-\frac{i\sqrt{11}}{3}}\operatorname{Tan}\left[\frac{1}{2}(-7-5i\sqrt{11})t-x\right] \tag{21}$$

Due to Eq (17), for b>0

$$u(x,t)=\left(\sqrt{-\frac{5}{3}-\frac{i\sqrt{11}}{3}}\right)\left(\left(\frac{1}{24}(\sqrt{3}\left(-5-i\sqrt{11}\right)^{\frac{3}{2}})+4\sqrt{3\left(-5-i\sqrt{11}\right)}\right)^{\frac{1}{2}}\right)$$
$$\tan\left[\left(\frac{1}{24}\sqrt{3}\left(-5-i\sqrt{11}\right)^{\frac{3}{2}}+4\sqrt{3\left(-5-i\sqrt{11}\right)}\right)^{\frac{1}{2}}\left(x-\frac{1}{2}\left(-7-5i\sqrt{11}\right)\mathrm{t}\right)\right] \tag{22}$$

Where b<0,

$$u(x,t)=\left(\sqrt{-\frac{5}{3}-\frac{i\sqrt{11}}{3}}\right)\left(\left(\frac{1}{24}\left(\sqrt{3}\left(-5-i\sqrt{11}\right)^{\frac{3}{2}}\right)+4\sqrt{3\left(-5-i\sqrt{11}\right)}\right)^{\frac{1}{2}}\right)$$
$$\tanh\left[\left(\frac{1}{24}\sqrt{3}\left(-5-i\sqrt{11}\right)^{\frac{3}{2}}+4\sqrt{3\left(-5-i\sqrt{11}\right)}\right)^{\frac{1}{2}}\left(x-\frac{1}{2}\left(-7-5i\sqrt{11}\right)\mathrm{t}\right)\right] \tag{23}$$

From Eq (18), then for b>0

$$u(x,t)=\left(-\sqrt{-\frac{5}{3}+\frac{i\sqrt{11}}{3}}\right)\left(\frac{1}{24}\left(-\sqrt{3}\left(-5+i\sqrt{11}\right)^{\frac{3}{2}}-4\sqrt{3\left(-5+i\sqrt{11}\right)}\right)\right)^{\frac{1}{2}}$$
$$\tan\left[\left(\frac{1}{24}\left(-\sqrt{3}\left(-5+i\sqrt{11}\right)^{\frac{3}{2}}-4\sqrt{3\left(-5+i\sqrt{11}\right)}\right)\right)^{\frac{1}{2}}\left(\mathrm{x}-\frac{1}{2}(-7+5i\sqrt{11})t\right)\right] \tag{24}$$

Where b<0,

$$u(x,t)=\left(-\sqrt{-\frac{5}{3}+\frac{i\sqrt{11}}{3}}\right)\left(\frac{1}{24}\left(-\sqrt{3}\left(-5+i\sqrt{11}\right)^{\frac{3}{2}}-4\sqrt{3\left(-5+i\sqrt{11}\right)}\right)\right)^{\frac{1}{2}}$$
$$\tanh\left[\left(\frac{1}{24}\left(-\sqrt{3}\left(-5+i\sqrt{11}\right)^{\frac{3}{2}}-4\sqrt{3\left(-5+i\sqrt{11}\right)}\right)\right)^{\frac{1}{2}}\left(\mathrm{x}-\frac{1}{2}\left(-7+5i\sqrt{11}\right)t\right)\right] \tag{25}$$

Eq (19) indicates that: for b>0,

$$u(x,t)=\left(\sqrt{-\frac{5}{3}+\frac{i\sqrt{11}}{3}}\right)\left(\frac{1}{24}\left(\sqrt{3}\left(-5+i\sqrt{11}\right)^{\frac{3}{2}}+4\sqrt{3\left(-5+i\sqrt{11}\right)}\right)\right)^{\frac{1}{2}}$$
$$\tan\left[\frac{1}{24}\left(\sqrt{3}\left(-5+i\sqrt{11}\right)^{\frac{3}{2}}+4\sqrt{3\left(-5+i\sqrt{11}\right)}\right)^{\frac{1}{2}}\left(\mathrm{x}-\frac{1}{2}\left(-7+5i\sqrt{11}\right)t\right)\right] \tag{26}$$

Where b<0

$$u(x,t)=\left(\sqrt{-\frac{5}{3}+\frac{i\sqrt{11}}{3}}\right)\left(\frac{1}{24}\left(\sqrt{3}\left(-5+i\sqrt{11}\right)^{\frac{3}{2}}+4\sqrt{3\left(-5+i\sqrt{11}\right)}\right)\right)^{\frac{1}{2}}$$
$$\tan h\left[\frac{1}{24}\left(\sqrt{3}\left(-5+i\sqrt{11}\right)^{\frac{3}{2}}+4\sqrt{3\left(-5+i\sqrt{11}\right)}\right)^{\frac{1}{2}}\left(\mathrm{x}-\frac{1}{2}\left(-7+5i\sqrt{11}\right)t\right)\right] \tag{27}$$

Basic idea of the Exp-function method [1]

We now present briefly the main steps of the Exp-function method that will be applied [4]. A traveling wave transformation $\xi = x - ct$ converts a partial differential Eq (5) in to Ordinary differential Eq (8)

The Exp-function method is based on the assumption that traveling wave solutions can be expressed in the following form:

$$u(\xi)=\frac{\sum_{n=-k}^{d}a_n\exp(n\xi)}{\sum_{m=-p}^{q}b_m\exp(m\xi)} \tag{28}$$

Where k, d, p are positive integers which are unknown to be further determined, a_n and b_m are known constants.

To determine the values of k and p and the values of d and q, we balance the linear term of the highest order in Eq (8) with the highest order nonlinear term respectively.

Application to the nonlinear fifth-order modified kdv equation

In order to obtain the solution of Eq (2) use the transformation $u = u(\xi)$, $\xi = x - ct$ so that Eq (2) becomes

$$\begin{aligned}&cu'(\xi)+30u(\xi)^4u'(\xi)+10u'(\xi)^2+10u(\xi)u''(\xi)\\&+20u(\xi)u'(\xi)u''(\xi)+10u(\xi)^2u^{(3)}(\xi)+u^{(5)}(\xi)=0\end{aligned} \tag{29}$$

Then by integrating Eq (29) and neglecting the constant of integration we obtain:

$$cu(\xi)+6u(\xi)^5+10u(\xi)u'(\xi)+10u(\xi)^2u''(\xi)+u^{(4)}(\xi) \tag{30}$$

According to the Exp-function method, we assume that the solution of Eq (30) can be expressed in the form

$$u(\xi)=\frac{\sum_{n=-k}^{d}a_n\exp(n\xi)}{\sum_{m=-p}^{q}b_m\exp(m\xi)} \tag{31}$$

Eq (31) can be written in an alternative form as follow

$$u(\xi)=\frac{a_k\exp(k\xi)+\ldots+a_d\exp(-d\xi)}{b_p\exp(p\xi)+\ldots+b_q\exp(-q\xi)} \tag{32}$$

In order to determined values of k and p we balance the nonlinear term of highest order in the Eq (31) with the highest order linear terms [6]. By simple calculation.

$$u^{(4)}(\xi)=\frac{C_1\exp[(k+15p)\xi]+\ldots}{C_2\exp[16p\xi]+\ldots} \tag{33}$$

$$u^5=\frac{C_3\exp[5k\xi]+\ldots}{C_4\exp[5p\xi]+\ldots} \tag{34}$$

Eq (33) becomes $u^{(4)}=\dfrac{C_1\exp[(k+4p)\xi]+\ldots}{C_2\exp[(5p)\xi]+\ldots}$ (35)

Where C_i are determined coefficients only for simplicity. Balancing highest order of Exp-function in Eq (34) with the highest order linear term we have by simply calculation

5K=k+4p, k=p (36)

Similarly to determine values of d and q, we balance the linear terms of the lowest order in Eq (32)

$$u^{(4)}=\frac{\ldots+d_1\exp[-(d+15q)\xi]+\ldots}{\ldots..+d_2\exp[-16q\xi]+\ldots} \tag{37}$$

$$u^5=\frac{\ldots.+d_3\exp[-(5d)\xi]+\ldots.}{\ldots..+d_4\exp[-5q\xi]+\ldots.} \tag{38}$$

We find

5d =d+4q, d=q (39)

Example 1: p=k=1 and d=q=1

$$u(\xi)=\frac{a_1\exp(\xi)+a_0+a_{-1}\exp(-\xi)}{b_1\exp(\xi)+b_0+b_{-1}\exp(-\xi)} \tag{40}$$

Substituting Eq (40) in Eq (30) and using Mathmatica software and equating the coefficients of exp ($k\xi$) to zero, we obtain a set of algebraic equations for a_0, a_1, a_{-1}, b_0, b_1, b_{-1} and c

$$\frac{1}{A}(C_1\exp(5\xi)+C_2\exp(4\xi)+C_3\exp(3\xi)+C_4\exp(2\xi)+C_5\exp(\xi)+C_0 \\ +C_{-1}\exp(-\xi)+C_{-2}\exp(-2\xi)+C_{-3}\exp(-3\xi)+C_{-4}\exp(-4\xi)+C_{-5}\exp(-5\xi)=0 \quad (41)$$

Where

$$A=\left(\left(b_{-1}+e^{\xi}(b_0+e^{\xi}b_1)\right)\right)^5$$

$$\begin{cases} C_1=0\ ,\ C_2=0,\ C_3=0\ ,\ C_4=0\ ,C_5=0 \\ C_0=0\ , \\ C_{-1}=0\ ,C_{-2}=0\ ,C_{-3}=0\ ,C_{-4}=0\ ,C_{-5}=0 \end{cases} \quad (42)$$

Solving the system of algebraic equations with the help of Mathmatica we get the following set of non-trivial solutions

$$a_0=0\ ,b_0=0\ ,\ b_{-1}=b_{-1},\ c=\frac{6a_{-1}^4}{b_{-1}^4}\ ,\ b_1=\frac{-a_1b_{-1}}{a_{-1}},\ a_{-1}=a_{-1}\ ,a_1=a_1 \quad (43)$$

$$u_1(x,t)=\frac{a_1\exp\left(x+\frac{6a_{-1}^4}{b_{-1}^4}t\right)+a_{-1}\exp(-x-\frac{6a_{-1}^4}{b_{-1}^4}t)}{b_{-1}\left[\frac{-a_1}{a_{-1}}\exp\left(x+\frac{6a_{-1}^4}{b_{-1}^4}t\right)+\exp\left(-x-\frac{6a_{-1}^4}{b_{-1}^4}t\right)\right]} \quad (44)$$

If we choose $a_{-1}=-1$, $a_1=1$ then

$$u_{1.1}=\frac{1}{b_{-1}}\tanh\left(x-\frac{6}{b_{-1}}t\right) \quad (45)$$

$$u_{1.2}=\frac{1}{b_{-1}}\coth\left(x-\frac{6}{b_{-1}}t\right) \quad (46)$$

Respectively, where a_1 and a_{-1} are arbitrary constants each of the obtained solitonary solutions can be converted in to a periodic solution.

Example 2: The seventh order modified kdv equation

We next apply the exp-function method to the seventh order mkdv equation which has the formula

$$u_t+\begin{Bmatrix} 20u^7+70\left(u^4u_{2x}+2u^3u_x^2\right)+14 \\ \left(u^2u_{4x}+3uu_{2x}^2+4uu_xu_{3x}+5u_x^2u_{2x}\right)+u_{6x} \end{Bmatrix}=0 \quad (47)$$

And by using the wave variable $\xi=x-ct$ reduce it to an ODE

$$cu'(\xi)+140u(\xi)^6u'(\xi)+420u(\xi)^2u'(\xi)^3+ \\ 560u(\xi)^3u'(\xi)u''(\xi)+182u'()u''(\xi)^2+70u(\xi)^4u^{(3)}(\xi)+ \\ 126u'(\xi)^2u^{(3)}(\xi)+140u(\xi)u''(\xi)u^{(3)}(\xi)+ \\ 84u(\xi)u'(\xi)u^{(4)}(\xi)+14u(\xi)^2u^{(5)}(\xi)+u^{(7)}(\xi)=0 \quad (48)$$

By integrating (48) and neglecting the constant of integration we obtain

$$cu(\xi)+20u(\xi)^7+140u(\xi)^3u'(\xi)^2+70(u(\xi)^4+u'(\xi)^2)u''(\xi)+42u(\xi)u''(\xi)^2 \\ +56u(\xi)u'()u^{(3)}(\xi)+14u(\xi)^2u^{(4)}()+u^{(6)}(\xi)=0 \quad (49)$$

In order to determine the values of K and p we balance $u^{(6)}$ with $u^{(6)}$ in Eq (49), to get

$$u^4u''=\frac{C_1\exp[(5k+3p)\xi]}{C_2\exp(8p\xi)} \quad (50)$$

$$u^{(6)}=\frac{C_3\exp[(k+63p)\xi]}{C_4\exp[(64p)\xi]} \quad (51)$$

5k+3p=k+7p (52)

Which leads to k=p, similarly to determine the values of d and q for the terms $u^{(6)}$ with u^4u''.

$$u^4u''=\frac{d_1\exp[(-5d-3q)\xi]}{d_2\exp[(-64q)\xi]} \quad (53)$$

$$u^{(6)}=\frac{d_3\exp[(-d-63q)\xi]}{d_4\exp[(-64q)\xi]} \quad (54)$$

-5d-3q=-d-7q (55)

Which leads to the result d=q

Now we consider the following cases:

Case 1 p=k=1 and q=d.

For simplicity, we p=k=1 and d=q=1. Then Eq (31) reduces to

$$u(\xi)=\frac{a_1\exp(\xi)+a_0+a_{-1}\exp(-\xi)}{b_1\exp(\xi)+b_0+b_{-1}\exp(-\xi)} \quad (56)$$

Substituting (56) in to Eq (51) and using Mathematica software, we have

$$\frac{1}{A}(C_1\exp(7\xi)+C_2\exp(6\xi)+C_3\exp(5\xi)+C_4\exp(4\xi)+C_5\exp(3\xi) \\ +C_6\exp(2\xi)+C_7\exp(\xi)+C_0+C_{-1}\exp(-\xi)+C_{-2}\exp(-2\xi)+C_{-3}\exp(-3\xi) \\ +C_{-4}\exp(-4\xi)+C_{-5}\exp(-5\xi)+C_{-6}\exp(-6\xi)+C_{-7}\exp(-7\xi)=0 \quad (57)$$

Where $A=\left(b_{-1}+e^{\xi}(b_0+e^{\xi}b_1)\right)^7$ (58)

And C_n are coefficients of exp(nx). Equating the coefficients of exp(nx) to zero, we obtain a set of algebraic equations for a_0, a_1, a_{-1}, b_0, b_1, b_{-1} and C. Solving the system of algebraic equations with the help of mathematica gives the following set of non-trivial solutions.

$$a_0=0,b_0=0,b_{-1}=\pm\sqrt{\frac{a_{-1}^2-i\sqrt{3}a_{-1}^2}{2}},b_1=\frac{a_1b_{-1}}{a_{-1}},C=2 \quad (59)$$

$$u_1(x,t)=\frac{a_1\exp(x-20t)+a_{-1}\exp(-x+20t)}{\frac{a_1b_{-1}}{a_{-1}}\exp(x-20t)+\sqrt{\frac{a_{-1}^2-i\sqrt{3}a_{-1}^2}{2}}\exp(-x+20t)} \quad (60)$$

$$a_0=0,b_0=0,b_{-1},b_1=\frac{a_1b_{-1}}{a_{-1}},a_1=a_1,a_{-1}=a_{-1},c=\frac{20a_{-1}^6}{b_{-1}^2} \quad (61)$$

$$u_2(x,t)=\frac{a_1\exp\left(x+\frac{20a_{-1}^6}{b_{-1}^2}t\right)+a_{-1}\exp\left(-x-\frac{20a_{-1}^6}{b_{-1}^2}t\right)}{\frac{a_1b_{-1}}{a_{-1}}\exp\left(x+\frac{20a_{-1}^6}{b_{-1}^2}t\right)+b_{-1}\exp\left(-x-\frac{20a_{-1}^6}{b_{-1}^2}t\right)} \quad (62)$$

$$a_0=0,b_0=0,a_1=a_1,b_1=\frac{b_{-1}a_1}{a_{-1}},a_{-1}=a_{-1},c=\frac{20a_{-1}^6}{b_{-1}^2}b_{-1}=\frac{-i\sqrt[6]{-5(-5i+7\sqrt{5})}a_{-1}}{\sqrt[6]{32}},c=\frac{64(5-7i\sqrt{5})}{135} \quad (63)$$

$$u_3(x,t)=\frac{a_1\exp\left(x-\frac{64(5-7i\sqrt{5})}{135}t\right)+a_{-1}\exp\left(-x+\frac{64(5-7i\sqrt{5})}{135}t\right)}{\frac{a_1\mathrm{b}_{-1}}{a_{-1}}\exp\left(x-\frac{64(5-7i\sqrt{5})}{135}t\right)+\frac{-i\sqrt[6]{-5(-5i+7\sqrt{5})}a_{-1}}{\sqrt[6]{32}}\exp\left(-x+\frac{64(5-7i\sqrt{5})}{135}t\right)} \quad (64)$$

Modified Kudryashov Method

Suppose we have a nonlinear evolution in the form Eq (5) we give the main steps of this method [9].

Using wave transformation ξ = x-ct to reduce ODE Eq (8).

Suppose that Eq (8) has the formal solution

$$u(\xi)=\sum_{n=i}^{N}a_i\,Q(\xi)^n \quad (65)$$

Where $a_n(n=0,1,2,\ldots..,N)$ are constants to be determined, such that $a_n\neq 0$ and Q(ξ) is a solution of the equation

$$Q'(\xi)=\left(Q(\xi)^2-Q(\xi)\right)\ln(a) \quad (66)$$

Equation (66) has the solution

$$Q(\xi)=\frac{1}{1\pm a^{\xi}} \quad (67)$$

We determine the positive integer N by balancing between the highest orders derivatives in linear terms with the highest order in nonlinear term in Eq (8).

Substitute equation (3.1) in to Eq (8) we calculate the necessary derivatives $u', u'', \ldots$ of the function u(x). As a result of this substitution we get a polynomial of $Q^i\,(i=0,1,2,\ldots)$ in this polynomial we gather all terms of same powers and equating them to zero, we obtain a system of algebraic equations which can be solved by mathematica to get un known parameters $a_n(n=0,1,2,\ldots)$ and c. Consequently we obtain the exact solution of equation (5).

Application modified kudryashov method on seventh order mkdv

The seventh order mkdv

$$u_t+\left\{20u^7+70\begin{pmatrix}u^2u_{2x}+2u^3u_x{}^2+14\\ \left(u^2u_{4x}+3uu_{2x}{}^2+4uu_xu_{3x}+5u_x{}^2u_{2x}\right)+u_{6x}\end{pmatrix}\right\}_x=0$$

Using wave transformation x= x-ct we have

$$\begin{aligned}&cu'(\xi)+140u(\xi)^6u'(\xi)+420u(\xi)^2u'(\xi)^3\\&+560u(\xi)^3u'(\xi)u''(\xi)+182u'(\xi)u''(\xi)^2+70u(\xi)^4u^{(3)}(\xi)\\&+126u'(\xi)^2u^{(3)}(\xi)+140u(\xi)u''(\xi)u^{(3)}(\xi)\\&+84u(\xi)u'(\xi)u^{(4)}(\xi)+14u(\xi)^2u^{(5)}(\xi)+u^{(7)}(\xi)=0\end{aligned} \quad (68)$$

We get N from by balancing between the highest order derivatives in linear terms with the highest order in nonlinear term in (65) we obtain N=1

$$u(\xi)=a_0+a_1\,Q(\xi) \quad (69)$$

Substituting in (68) and collecting coefficients of same powers and equating them to zero, we obtain a system of algebraic equations which can be solved by mathematica to get un known parameters a_0 , a_1 , c , we get

$$a_0=-\frac{1}{2}i\mathrm{Ln}(a),a_1=i\mathrm{Ln}(a),c=\frac{5\mathrm{Ln}(a)^6}{16} \quad (70)$$

$$a_0=\frac{1}{2}i\mathrm{Ln}(a),a_1=i\mathrm{Ln}(a),c=\frac{5\mathrm{Ln}(a)^6}{16} \quad (71)$$

From (70) and (71) we obtain

$$u_1(x,t)=-\frac{1}{2}i\ln(a)+\frac{i\ln(a)}{1\pm a^{x+\frac{5}{16}\ln(a)^6}} \quad (72)$$

$$u_2(x,t)=\frac{1}{2}i\ln(a)-\frac{i\ln(a)}{1\pm a^{x+\frac{5}{16}\ln(a)^6}} \quad (73)$$

Physical Explanations of our Obtained Solutions

Solitary bell-type waves have been obtained. In this section we have presented some graphs of these solutions by taking suitable values of involved unknown parameters to visualize the underlying mechanism of the original equations. Using mathematical software Maple or Mathematica, the plots of some obtained solutions of Eqs (72), (73), (20) and (23) have been shown in Figures 1-4.

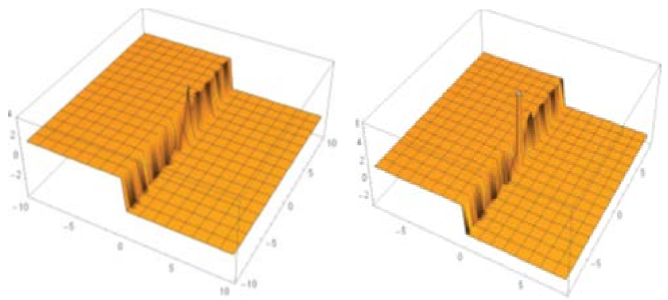

Figure 1: The plot of the solutions (72), when a=2.

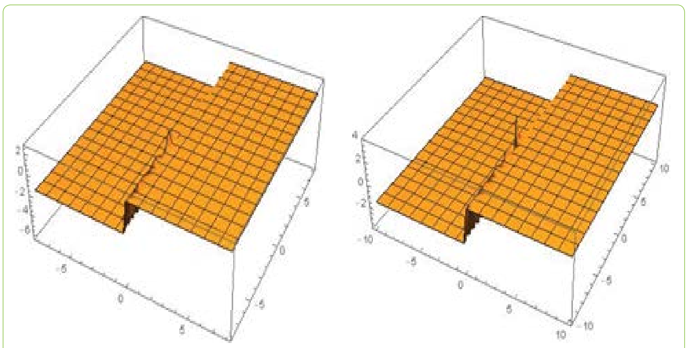

Figure 2: The plot of the solutions (73), when a=2.

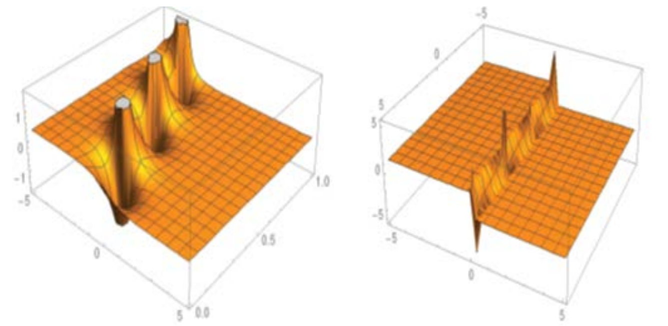

Figure 3: The plot of the solutions (20) and (21).

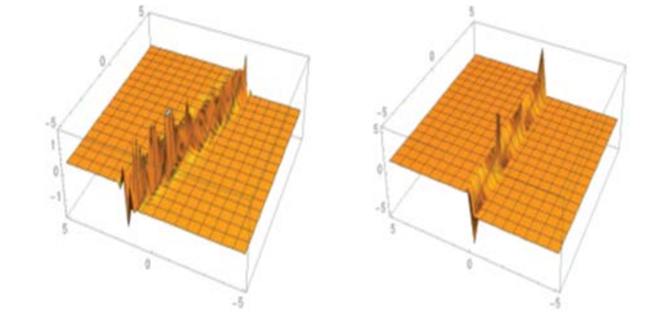

Figure 4: The plot of the solutions (22) and (23).

Conclusion

We have presented the tanh method, exp-function method, kudryshove method and homogenous balance method to solve the higher order modified kdv equations. The tanh-method and exp-function method are powerful in searching for exact solutions of NLPDEs. We find that these methods are have been successfully applied to obtain some new generalized solitonary solutions to the modified kdv equations and this prove that these methods are efficient technique in finding exact solutions for wide classes of problems.

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